13.1 -

1. $\vec{AB} = \langle 2, 3 \rangle$

   $\vec{PQ} = \langle 6, 9 \rangle$

2. $\vec{OR} = \langle -2, 7 \rangle \quad \|OR\| = \sqrt{4+49} = \sqrt{53}$

3. $\langle 14, 8 \rangle$

13.2 -

2. $\langle 8, 12, -6 \rangle = 2\langle 4, 6, -3 \rangle$ same dir ✓

   $r(0) = \langle 3, -1, 4 \rangle = s(-2)$ same pt. ✓

3. $r_1(t) = \langle 2-4t, 1, 1+t \rangle$

   $r_2(s) = \langle -4+2s, 1+s, 5-2s \rangle$

$\underline{y}$: $1 = 1+s \Rightarrow s = 0$

$\underline{x}$: $2-4t = -4+2s = -4 \Rightarrow t = {}^{-6}/_{-4} = {}^{3}/_{2}$

$\underline{z}$: $1 + \frac{3}{2} \neq 5$ No intersection

13.3 –

(1) $\theta = \cos^{-1}\left(\dfrac{\langle 3,1,1\rangle \cdot \langle 2,-4,2\rangle}{\sqrt{9+1+1}\cdot\sqrt{4+16+4}}\right)$ $\sqrt{}$

$= \cos^{-1}\left(\dfrac{6-4+2}{\sqrt{11}\sqrt{24}}\right) = \cos^{-1}\left(\dfrac{4}{\sqrt{264}}\right)$

(2) $\cos\theta_{u,v} = \dfrac{u\cdot v}{\|u\|\|v\|} = \dfrac{e_v\cdot v + e_w\cdot v}{\sqrt{2}\ \|v\|}$

$= \dfrac{\cancel{\|v\|} + \cancel{\|v\|}\cos\theta_{v,w}}{\sqrt{2}\ \cancel{\|v\|}}$

$= \dfrac{1+\cos\theta_{v,w}}{\sqrt{2}}$

$\cos\theta_{u,w} = \dfrac{u\cdot w}{\|u\|\|w\|} = \dfrac{\cancel{\|w\|}\cos\theta_{v,w} + \cancel{\|w\|}}{\sqrt{2}\ \cancel{\|w\|}}$

$= \dfrac{1+\cos\theta_{u,w}}{\sqrt{2}} = \cos\theta_{u,v}$ ✓

3 No, for ex $i \cdot k = j \cdot k = 0$

but $i \neq j$

13.4

1 a) $3\, u \times w + \cancel{4\, w \times w}^{0}$

$= 3 \langle 0, 3, 1 \rangle$

b) $\cancel{u \times u}^{0} - u \times v + v \times u - \cancel{v \times v}^{0}$

$= 2\, v \times u = -2 \langle 1, 1, 0 \rangle$

2 $i \times j = k, \quad j \times k = i, \quad k \times i = j$

3 $$|u \cdot (v \times w)| = \begin{vmatrix} 2 & 2 & 1 \\ 1 & 0 & 3 \\ 0 & -4 & 0 \end{vmatrix}$$

$$= 4 \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} = 4(6-1) = \boxed{20}$$

13.5

① $x + 3y + 2z = d$

$4 - 3 + 2 = d \Rightarrow d = 3$

$\boxed{x + 3y + 2z = 3}$

② $\overrightarrow{PQ} = \langle -1, 2, -3 \rangle$

$\overrightarrow{PR} = \langle 1, 2, -6 \rangle$

$$PQ \times PR = \begin{vmatrix} i & j & k \\ -1 & 2 & -3 \\ 1 & 2 & -6 \end{vmatrix}$$

$$= i(-12+6) - j(6+3) + k(-2-2)$$

$$= -6i - 9j - 4k = \langle -6, -9, -4 \rangle$$

$-6x - 9y - 4z = d$

Plug in: $(1,1,1) \Rightarrow -6-9-4 = d \Rightarrow \boxed{d = -19}$

$$\boxed{-6x - 9y - 4z = -19}$$

3 $r(t) = \langle 1, 1+2t, 4t \rangle$

$1 + 1 + 2t + 4t = 14$

$\Rightarrow \quad 6t = 12 \quad \Rightarrow \quad t = 2$

$r(2) = \boxed{(1, 5, 8)}$

<u>14.1 -</u>

1 $\sin t = 0 \quad \Rightarrow \quad t = K\pi$

$\cos t/2 = 0 \quad \Rightarrow \quad \frac{t}{2} = K\frac{\pi}{2}$

<u>Yes</u>, when t is a multiple of π

2 <u>center</u>: $(7, 0, 0)$

<u>radius</u>: 12

14.2 -

① $\frac{dr_1}{dt} = \langle 2t, 3t^2, 1 \rangle$

$\frac{dr_2}{dt} = \langle 3e^{3t}, 2e^{2t}, e^t \rangle$

a. $\frac{dr_1}{dt} \cdot r_2 + r_1 \cdot \frac{dr_2}{dt}$

$= 2te^{3t} + 3t^2e^{2t} + e^t + 3t^2e^{3t} +$
$2t^3e^{2t} + te^t$

$= \boxed{(2t + 3t^2)e^{3t} + (3t^2 + 2t^3)e^{2t} + (1+t)e^t}$

b. $\langle t^3e^t + 3t^2e^t - 2te^{2t} - e^{2t},$
$3te^{3t} + e^{3t} - t^2e^t - 2te^t,$
$2t^2e^{2t} + 2te^{2t} - 3t^3e^{3t} - 3t^2e^{3t} \rangle$

(2) $$\int_{-2}^{2} \langle t^2+4t,\ 4t^3-t \rangle \, dt$$

$$= \left\langle \frac{t^3}{3}+2t^2,\ t^4-\frac{t^2}{2} \right\rangle \Big|_{-2}^{2}$$

$$= \left\langle \frac{8}{3}+8,\ 16-2 \right\rangle - \left\langle -\frac{8}{3}+8,\ 16-2 \right\rangle$$

$$= \left\langle \frac{16}{3},\ 0 \right\rangle$$

14.3 -

(1) $$\int_1^4 \|r'(t)\| \, dt = \int_1^4 \left\| \left\langle 2, \frac{1}{t}, 2t \right\rangle \right\| dt$$

$$= \int_1^4 \sqrt{4+\frac{1}{t^2}+4t^2} \, dt = \int_1^4 \sqrt{\frac{4t^4+4t^2+1}{t^2}} \, dt$$

$$= \int_1^4 \sqrt{\frac{(2t+1)^2}{t^2}} \, dt = \int_1^4 \frac{2t+1}{t} \, dt$$

$$= \int_1^4 2 + \frac{1}{t}\, dt = 2t + \ln|t| \Big|_1^4$$

$$= 8 + \ln(8) - 2 - 0 = \boxed{6 + \ln(8)}$$

(2)

$$s(t) = \int_0^t \| \langle 2u, 4u, 3u^2 \rangle \| \, du$$

$$= \int_0^t \sqrt{4u^2 + 16u^2 + 9u^4}\, du$$

$$= \int_0^t u\sqrt{20 + 9u^2}\, du$$

$$= \frac{(9u^2 + 20)^{3/2}}{27} \Big|_0^t$$

$$= \frac{(9t^2 + 20)^{3/2}}{20} - \frac{(20)^{3/2}}{27}$$

③ a) $s = g(t) = \int_0^t \| \langle 3, 4, 2 \rangle \| \, du$

$$= \int_0^t \sqrt{9+16+4} \, du = \sqrt{29}\, t$$

$$g^{-1} = \frac{s}{\sqrt{29}}$$

$$r(g^{-1}(s)) = \left\langle \frac{3s}{\sqrt{29}} + 1, \frac{4s}{\sqrt{29}} - s, \frac{2s}{\sqrt{29}} \right\rangle$$

b) $s = g(t) = \int_0^t \| \langle e^u \cos u + e^u \sin u, -e^u \sin u + e^u \cos u, e^u \rangle \| \, du$

$$= \int_0^t e^u \sqrt{\cos^2 + \cancel{2\cos\sin} + \sin^2 + \sin^2 \cancel{- 2\sin\cos} + \cos^2 + 1} \, du$$

$$= \int_0^t e^u \sqrt{3} \, du = \sqrt{3} e^u \Big|_0^t = \sqrt{3} e^t - \sqrt{3}$$

$$s + \sqrt{3} = \sqrt{3} e^t \Rightarrow t = \ln\left(\frac{s+\sqrt{3}}{\sqrt{3}}\right) = g^{-1}(s)$$

$$r(g^{-1}(s)) = \frac{s+\sqrt{3}}{\sqrt{3}} \left\langle \sin\left(\ln\left(\frac{s+\sqrt{3}}{\sqrt{3}}\right)\right), \cos\left(\ln\left(\frac{s+\sqrt{3}}{\sqrt{3}}\right)\right), 1 \right\rangle$$